VECTOR ANALYSIS

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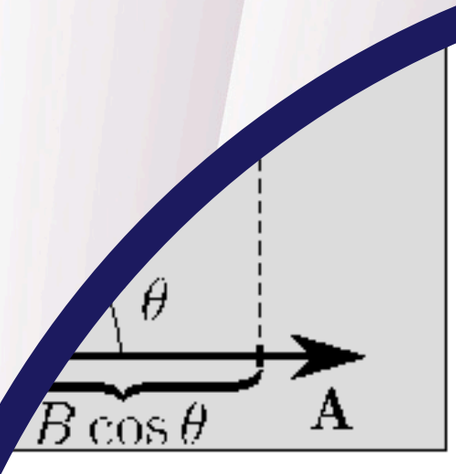
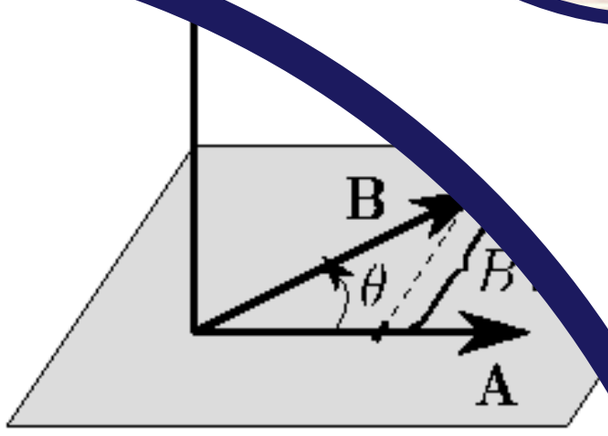
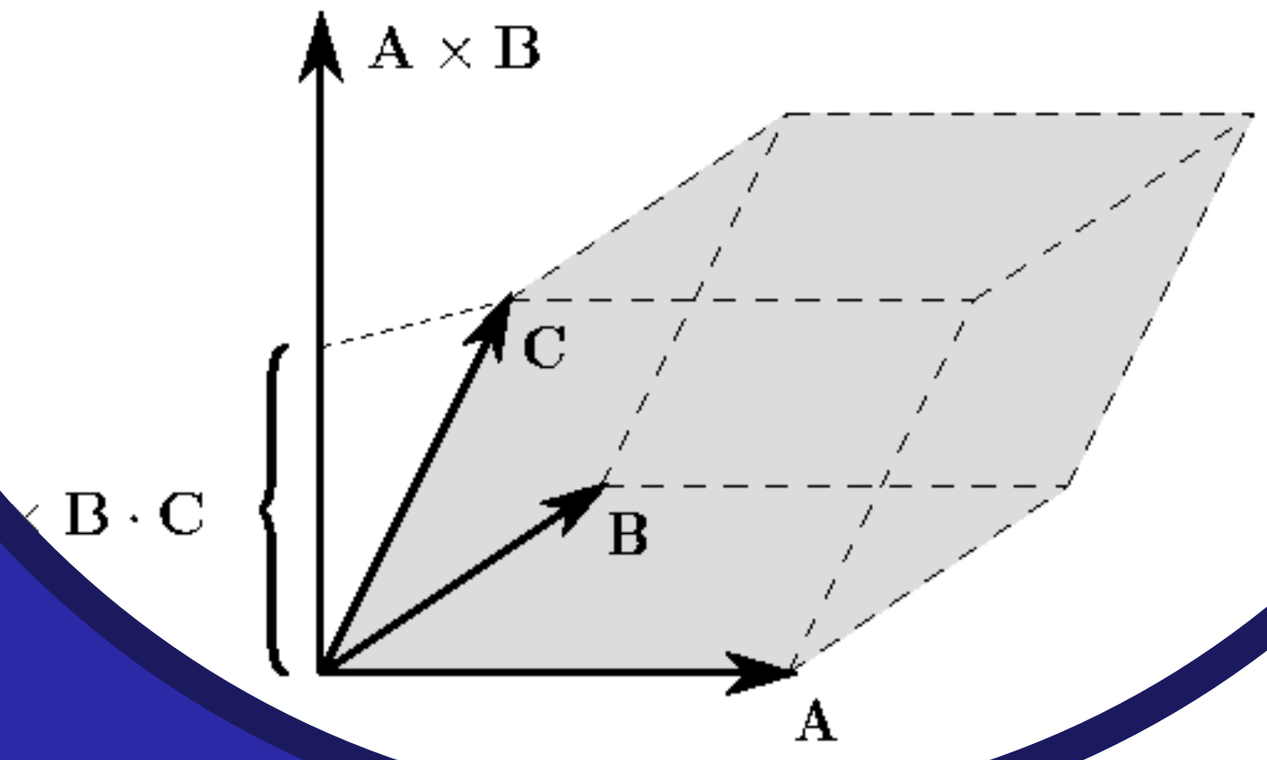


$$\mathbf{A} \cdot \mathbf{B} = AB\cos\theta$$



$$\mathbf{A} \times \mathbf{B} = \hat{\mathbf{c}}_{\mathbf{A}\times\mathbf{B}} AB\sin\theta$$

dot-cross-product



Vector Analysis Numerical Problems

Vector Addition and Subtraction

1. Two vectors A = 3i + 4j and B = 1i - 2j are given. Find A + B.

2. Find the magnitude of the resultant vector when A = 5i + 12j and B = -3i + 4j are added.

3. Given A = 7i + 24j and B = -7i - 24j, find A - B.

4. Calculate the resultant vector when A = 10i + 2j and B = 3i + 6j are added.

5. Find the unit vector in the direction of the vector A = 8i - 6j.

Dot Product

6. Calculate the dot product of vectors A = 4i + 3j and B = -2i + j.

7. Find the angle between the vectors A = i + j and B = i - j.

8. Given vectors A = 3i + 4j and B = 4i - 3j, determine if they are perpendicular.

9. Calculate the projection of vector A = 5i + 2j onto vector B = i + 2j.

10. If A = 2i + 2j and B = 4i + 3j, find A · B.

Cross Product

11. Find the cross product of vectors A = 2i + 3j and B = i - 4j.

12. Calculate the area of the parallelogram formed by vectors A = i + j and B = i - j.

13. Determine the cross product of A = i + 2j + k and B = i - j.

14. If A = 2i + 3j - k and B = i + 2j + k, find A × B.

15. Given A = 3i - j + 2k and B = i + j + k, calculate A × B.

Vector Components

16. Resolve the vector A = 10i + 10j into components along i and j.

Vector Analysis Numerical Problems

17. A vector A has a magnitude of 5 units and makes an angle of 30° with the positive x-axis. Find its components.

18. Given a vector A in the xy-plane with a magnitude of 7 units and making an angle of 45° with the x-axis, determine its x and y components.

19. If A = 6i + 8j, what is the magnitude of A and its direction?

20. Resolve A = 3i - 4j into its perpendicular components.

Vector Magnitude and Direction

21. Find the magnitude and direction of the vector A = 7i + 24j.

22. Given A = -6i + 2j, find the magnitude and the angle it makes with the positive x-axis.

23. Calculate the magnitude of the vector A = 12i + 5j - 9k.

24. Find the direction cosines of the vector A = 2i + 3j + 6k.

25. Determine the unit vector in the direction of A = 4i - j + 7k.

Applications in Physics

26. A force F = 10i + 20j N is applied to a particle. Find the work done when the particle moves from point (0,0) to (2,3) m.

27. A displacement vector is given by d = 5i + 4j meters. Find the work done by a force F = 3i + j N along this displacement.

28. If the velocity vector of a particle is v = 3i + 4j m/s, find its speed.

29. A particle moves under the influence of a constant force F = 6i + 8j N. Find the acceleration if the mass of the particle is 2 kg.

30. Given the position vector r(t) = ti + t^2j, find the velocity and acceleration vectors.

Vector Analysis Numerical Problems

Vector Equations

31. Solve for A if A + B = 5i + 7j and B = 2i + 3j.

32. Find vector C such that A + B + C = 0 given A = 4i + j and B = i - 2j.

33. If A = xi + yj and B = 3i + 4j are perpendicular, find the values of x and y.

34. Given A = 2i + 3j + 4k and B = i - j + k, solve for the scalar k such that A + kB = 0.

35. Find the vector A if 3A - 2B = 4i - j and B = i + j.

Vector Projections

36. Find the projection of vector A = 3i + 4j onto vector B = i + 2j.

37. Given vectors A = 5i + 4j and B = 2i + 3j, calculate the projection of A onto B.

38. Calculate the magnitude of the vector C = 4i + 3j + 5k.

39. Find the angle between vectors D = 2i + 2j and E = i + 3j.

40. If F = 3i + 4j and G = i + 2j, determine the angle between them using the dot product.

41. A force vector H = i + 2j + k is applied to a particle. Find the work done if the particle moves along a displacement vector r = 2i + j + 3k.

42. Given vectors I = 6i + 8j and J = 3i + 4j, check if they are parallel.

43. Calculate the torque r x F, where r = 2i + 3j + k and F = i + j + k.

44. Find the resultant of three vectors A = 3i + 4j, B = -i + 2j, and C = 5i - j.

45. Determine the scalar triple product of vectors A = i + j, B = j + k, and C = k + i.

46. Given a velocity vector v = 4i + 3j + k, find its magnitude and direction.

47. Find the unit vector perpendicular to both vectors A = 2i - j + k and B = i + j - k.

48. A force F = 10i - 5j + 2k acts on a particle. Find the component of this force along the vector G = i + k.